RMSC 4003
Statistical Modeling in Financial Markets
Tutorial 4 Solution

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This tutorial is for your reference on using R to solve Question 4 in Homework 2.

Example 0.1. (a) Using the data in the data file, calculate the annual covariance matrix and the annual expected rates of returns of these stocks.

(b) Assume, for simplicity, the listed stocks below form the market portfolio. Based on the Example 6.11 in the textbook (Investment Science), calculate the efficient market weights w_1, w_2 of the two efficient portfolios with short-selling. That is:

(1) Solve for the weights when $\lambda_1 = 1, \lambda_2 = 0$ and then normalize the weights.

(2) Solve for the weights when $\lambda_1 = 0, \lambda_2 = 1$ and then normalize the weights.

Using the two-fund theorem, determine the efficient market portfolio volatility σ and the market portfolio expected rate of return $\bar{r}$ when $\alpha = 0.5$, where the new efficient portfolio weight is $\alpha w_1 + (1-\alpha)w_2$.

(c) Using the two-fund theorem, plot the efficient frontier of the market portfolio (in part (b)) with short-selling. Suppose the risk-free interest rate $r_f = 3\%$. Determine the efficient frontier when lending and borrowing of the risk-free asset is allowed. Locate on the diagram also the point of unique risky fund and draw the efficient frontier.

Solution. (a)
```
# read the data
data = read.csv("tutorial_4_data.csv")
price = data[,2:5]

# calculate the return of the stocks
return = (data[2:7,2:5] - data[1:6,2:5])/data[1:6,2:5]

# annual expected rates of returns
apply(return,2,mean)

# annual covariance matrix
cov(return)
```

(b)
```
# lambda_1 = 1, lambda = 0
w1 = solve(cov(return),apply(return,2,mean))

# lambda_1 =0, lambda = 2
```

```
w2 = solve(cov(return),c(1,1,1,1))

# Normalized weights
norm_w1 = w1/sum(w1)
norm_w2 = w2/sum(w2)

# form the new portfolio
new_w = 0.5*norm_w1 + 0.5*norm_w2

# mean of the new efficient portfolio
new_w %*% apply(return,2,mean)
# sigma of the new efficient portfolio
sqrt(t(new_w)%*%cov(return)%*%new_w)
```

(c)
```
# note that norm_w2 corresponds to the global minimum variance point

# Plot the minimum variance set
alpha = seq(-9.98,10,length.out=1000)
two_weight = matrix(c(1:4000),ncol=4)
for (i in 1:1000) {
 two_weight[i,] = alpha[i]*norm_w2 + (1-alpha[i])*norm_w1
}
mean_eff = two_weight %*% apply(return,2,mean)
sigma_eff = sqrt(diag(two_weight%*%cov(return)%*%t(two_weight)))
plot(sigma_eff,mean_eff,cex=0.1,xlim=c(0,0.08),ylim=c(0,0.15),
main="Efficient Frontier (Blue) and Minimum Variance Set",
xlab=expression(sigma),ylab=expression(mu))
# plot the efficient frontier
points(sigma_eff[1:550],mean_eff[1:550],col="blue",cex=0.1)

# find the tangency portfolio
v = solve(cov(return), apply(return,2,mean)-0.03)
w = v/sum(v)

# add the tangency portfolio on the graph and the line
# connecting the risk free asset
# and the tangency portfolio
mean_t = w%*% apply(return,2,mean)
sigma_t = sqrt(t(w)%*% cov(return)%*%w)
points(sigma_t,mean_t,col="red",pch=16)
abline(0.03,(mean_t-0.03)/sigma_t)
```

$$\text{Mean} = (0.1537, 0.1117, 0.0140, 0.0028).$$

$$\text{Covariance matrix} = \begin{pmatrix} 0.0555 & 0.1036 & -0.0358 & 0.0294 \\ 0.1036 & 0.2102 & -0.0596 & 0.0655 \\ -0.0358 & -0.0596 & 0.0350 & -0.0120 \\ 0.0294 & 0.0655 & -0.0120 & 0.0541 \end{pmatrix}.$$

$\text{normw}_1 = (0.9689, -0.3508, 0.3960, -0.0141)$.
$\text{normw}_2 = (0.8427, -0.2939, 0.4248, 0.0263)$.
New efficient portfolio weight $= (0.9058, -0.3223, 0.4104, 0.0061)$ with $\mu = 0.1090$ and $\sigma = 0.04432$.
Tangency portfolio $= (1.0209, -0.3743, 0.3841, -0.0307)$.